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Optimal algorithm to Solve Transportation problem with Hexagonal Fuzzy Numbers

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ABSTRACT
The present study focus is to deal with transportation problem. It is one of crucial and important topic that helps to optimize the transportation cost, time, space etc. Here, a new algorithm is proposed to find optimal solutions to the fuzzy transportation problem. Further, ranking methodology is used to defuzzify the hexagonal fuzzy numbers to get optimal solution. The results of the proposed technique give best optimal solution as compared with existing algorithms. Finally, the validity of the proposed method is illustrated with the numerical example. We recommend that, proposed algorithm will be fruitful for dealing with similar kind of study.

KEYWORDS
Transportation; fuzzy; optimality; ranking; Hexagonal fuzzy numbers

1. Introduction

Operations Research is a state of art approach used for problem-solving and decision making. It helps any organization to achieve their best performance under the given constraints or situations. The transportation problem is one of the special areas found worldwide which is helpful to solve real life problems. Transportation problems play an important role in production, distribution etc. purposes and are a special case of linear programming problems. It helps in minimizing the cost function. There are numerous things that decide the cost of transport. It includes the distance between the two locations, the path followed, mode of transport, the number of units that are transported, the speed of transport, etc. The main focus is to transport the commodities with minimum transportation cost without any compromise in supply and demand. The transportation model can also be used in making location decisions. There are several research studies to bring the best optimal solution to the transportation problems.The problem that needs to be resolved is, to reach cost effective production in various production companies. [1-19] Bagheri et al. (2020) investigated a transportation problem with fuzzy costs in the presence of multiple and conflicting objectives

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using fuzzy data envelopment analysis approach. Kacher and Singh (2021) performed a systematic and organized overview of various existing transportation problems and their extensions developed by different researchers. The main purpose of the review paper is to recapitulate the existing form of various types of transportation problems and their systematic developments for the guidance of future researchers to help them classify the varieties of problems to be solved and select the criteria to be optimized. Bagheri and Behnamian (2022) performed a systematic literature review on the multi-factory scheduling problems in the past eleven years and reported research gaps. Polat and Topaloğlu (2022) proposed a novel mathematical model for the collection of different types of milk from producers by multi-tank tankers with split deliveries, uncertain demand, service time and vehicle speed conditions. A real-life case study from a dairy company is solved under different risk assessment scenarios. Indeed, several brand-new benchmark instances for the core problem are presented and solved by utilizing an efficient heuristics approach called enhanced iterative local search. Their findings of the study showed that logistics decision makers should design their collection networks with low, but non-zero, risk levels. Sleiman et al. (2022) established a control strategy in order to prevent the congestion in the transportation network subject to additive uncertainties. Qamsari et al. (2022) proposed a novel approach towards inventory routing problems with fuzzy time windows considering customer satisfaction for arrival intervals. Their study divided into three categories according to their features. These features have different degrees of importance from the distributers' perspective. The satisfaction level of customers plays an important role in the decision of the supplier to fulfil their demand in each period. Finally, they have studied and proposed a real-world case study of a blood distribution system model for Tehran. Verma (2022)studied fuzzy shortest path problem provides the shortest way to the decision-maker having least possible distance from source to destination. They proposed complete ranking method to solve the fuzzy shortest path problem. The proposed approaches ensure that the fuzzy shortest distance is equal among all possible shortest paths. Bharati (2022) defines the hesitant fuzzy membership function and non-membership function to tackle the uncertainty and hesitation of the parameters. A new solution called hesitant intuitionistic fuzzy Pareto optimal solution is defined, and some theorems are stated and proved. Taghavi et al. (2022) proposed a new model for the bus terminal location problem using data envelopment analysis with multi-objective programming approach. Their focus is on to find efficient allocation patterns for assigning stations terminals. Also, they investigated the optimal locations for deploying terminals using a genetic algorithm for solving proposed model. Bera and Mondal (2022) dealt with multi-objective transportation problem under cost-dependent credit period policy. Here, the items are transported from a production house to the retailers by distributors who act as mediators. For adequate uncertainty in the cost parameter, it is considered as Gaussian fuzzy number. The effect of fuzziness on the model solution has been analyzed to improve their profit structure and retailers can reduce their cost structure under the used cost dependent credit period policy.

1.1. Definitions

- **Fuzzy set:** A fuzzy set $\tilde{A}$ on $\mathbb{R}$ is defined as a set of ordered pairs

$$\tilde{A} = \left\{ (X_0, \mu_A(X_0)) \mid X_0 \in \tilde{A}, \mu_A(X_0) \in [0,1] \right\},$$

where $\mu_A(X_0)$ is said to be the membership function.

- **Fuzzy number:** A fuzzy set $\tilde{A}$ on $\mathbb{R}$ is called a fuzzy number if it satisfies the following conditions:
  - $\mu_A(X_0)$ is continuous,
  - There exists at least one $X_0 \in \mathbb{R}$ with $\mu_A(X_0) = 1$,
  - $\tilde{A}$ is regular and convex.
- **Hexagonal fuzzy number:** A fuzzy number $\tilde{A}$ on $\mathbb{R}$ is called a hexagonal fuzzy number (or linear number) and is denoted as $(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5, \tilde{a}_6)$ if its membership function $\mu_A(X)$ has the following characteristics:

$$\mu_A(X) = \begin{cases} 0, & X < \tilde{a}_1 \\ \frac{1}{2}\frac{X-\tilde{a}_1}{\tilde{a}_2-\tilde{a}_1}, & \tilde{a}_1 \le X \le \tilde{a}_2 \\ \frac{1}{2} + \frac{1}{2}\frac{X-\tilde{a}_2}{\tilde{a}_3-\tilde{a}_2}, & \tilde{a}_2 \le X \le \tilde{a}_3 \\ 1, & \tilde{a}_3 \le X \le \tilde{a}_4 \\ 1 - \frac{1}{2}\frac{X-\tilde{a}_4}{\tilde{a}_5-\tilde{a}_4}, & \tilde{a}_4 \le X \le \tilde{a}_5 \\ \frac{1}{2}\frac{\tilde{a}_6-X}{\tilde{a}_6-\tilde{a}_5}, & \tilde{a}_5 \le X \le \tilde{a}_6 \\ 0, & X > \tilde{a}_6 \end{cases}$$

2. Methodology

2.1. Algorithm

Step 1: Convert the given hexagonal fuzzy numbers to crisp number using following function:

$$R(\tilde{a}) = \frac{2a_1 + 3a_2 + 4a_3 + 4a_4 + 3a_5 + 2a_6}{18} \tag{1}$$

Step 2: Check whether the given transportation problem is balanced or unbalanced.
2.1: If it is balanced, then go to step 3.
2.2: If it is unbalanced, then add a dummy row or dummy column to fulfil the requirement.
Step 3: Find the First minimum and maximum of each row and take the product of them and the product is divided by the product of number of rows and number of columns in the given table of respective iteration.
Step 4: Find the First minimum and maximum of each column and take the product of them and the product is divided by the product of number of rows and number of columns in the given of respective iteration.
Step 5: After simplifying step 3 and 4, select the largest ratio and allocate as much as possible to the smallest element in the respective row (column) to fulfil the demand or to exhaust the availability.
Step6: If maximum ratio value may occur more than once in the rows or columns then arbitrarily select any one row or column but not both.
Step 7: Repeat the step 3 to 6, until all the availability and demand will get exhausted or fulfilled.
Step 8: To check if the number of allocations is m+n-1 or not. If it is less than m+n-1, then apply the MODI method to check the optimality of the given problem.

3. Result and discussion

Numerical example: A resolution that affirms the fuzzy transportation problem which involves transportation cost, customer needs and demands and existence of products using pentagonal fuzzy numbers. Observe the following transportation problem as stated in Table 1 [12]:

Table 1. Given Dataset

	D_1	D_2	D_3	D_4	Availability
O_1	(3,7,11,15,19,24)	(13,18,23,28,33,40)	(6,13,20,28,36,45)	(15,20,25,31,38,45)	(7,9,11,13,16,20)
O_2	(16,19,24,29,34,39)	(3,5,7,9,10,12)	(5,7,10,13,17,21)	(20,23,26,30,35,40)	(6,8,11,14,19,25)
O_3	(11,14,17,21,25,30)	(7,9,11,14,18,22)	(2,3,4,6,7,9)	(5,7,8,11,14,17)	(9,11,13,15,18,20)
Demand	(3,4,5,6,8,10)	(3,5,7,9,12,15)	(6,7,9,11,13,16)	(10,12,14,16,20,24)	

Table 2. Defuzzified Transportation problem

	D_1	D_2	D_3	D_4	Availability
O_1	13.11	25.72	69.5	28.77	12.5
O_2	26.72	7.72	12	28.77	13.5
O_3	19.5	13.27	5.11	10.94	14.28
Demand	5.89	8.39	10.22	15.78	

Table 3. First Iteration

	D_1	D_2	D_3	D_4	Availability	Min*Max/ Row*Column
O_1	**5.89** 13.11	25.72	69.5	28.77	12.5	← 75.92
O_2	26.72	7.72	12	28.77	13.5	18.5087
O_3	19.5	13.27	5.11	10.94	14.28	8.3037
Demand	5.89	8.39	10.22	15.78		
Min*Max/ Row*Column	29.224	16.546	29.59	26.228		

Total Minimum cost= 5.8913.11+6.6125.72+10.9414.28+1.787.72+10.2212+1.5 28.77

Total Minimum cost= 582.9869

Table 4. Second Iteration

	D_2	D_3	D_4	Availability	Min*Max/ Row*Column
O_1	**6.61** 25.72	69.5	28.77	← 6.61	198.615
O_2	7.72	12	28.77	13.5	24.678
O_3	13.27	5.11	10.94	14.28	7.5344
Demand	8.39	10.22	15.78		
Min*Max/ Row*Column	22.06	39.46	34.971		

Table 5. Third Iteration

	D_2	D_3	D_4	Availability	Min*Max/ Row*Column
O_2	7.72	12	28.77	13.5	37.0174
O_3	13.27	5.11	**14.28** 10.94	14.28	11.3016
Demand	8.39	10.22	15.78		
Min*Max/ Row*Column	17.074	10.22↑	52.4573		

Table 6. Final Iteration

	D_2	D_3	D_4	Availability
O_2	**1.78** 7.72	**10.22** 12	**1.5** 28.77	13.5
Demand	8.39	10.22	15.78	

4. Conclusions

In this study, the proposed algorithm gives the best possible viability of the fuzzy transportation problem for hexagonal fuzzy numbers. The comparison of the proposed method with ranking method [14] and is tabulated below (Table 7):

Table 7. Comparative results

Methods	Optimal Solutions
Ranking method [14]	7618
Proposed method	582.9869

In general, this algorithm can be useful for all types of fuzzy transportation problems. This approach could be generalized to resolve similar kinds of transportation problems. The proposed algorithm helps in locating a new facility, a manufacturing plant or an office when two or more locations are under consideration. In a nutshell, the total transportation cost, distribution cost or shipping cost and production costs are to be minimized by applying the model to the similar kind of studies.

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